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Two-phase steady flow along a horizontal glass pipe in the presence of the magnetic and electrical fields

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Abstract

This study examines the effect of the electrical and magnetic field which is applied perpendicular to the flow and each other on the two-phase steady flow in a glass pipe. Micron-sized iron powder, which are highly conductive, magnetizable, are used for the first phase of the fluid and then pure water which is not magnetizable and has very low electrical conductivity is used for the second phase. The mathematical model is derived by adding a term representing the impact of electro-magnetic force to the momentum equation of the multiphase fluids in the interactive magnetizable phase. The derived model is analytically solved by using the methods of Laplace and D'Alambert. According to obtained results, when only magnetic and electrical fields are applied perpendicular to the flow of the mixture, local flow velocity of the first phase is decreased due to the direct effect of the magnetic field. The second phase local flow velocity is decreased due to the indirect effect of the magnetic field which is caused by the interaction of the phases. As a result, it is seen that the electromagnetic force is effecting the nonconductor phase of the mixture through the conductor phase which it can directly affect.

Keywords: Two-phase flow; Steady flow; Glass pipe; Electrical field; Magnetic field

1. Introduction

Two-phase flow of fluid–solid is used in many fields like mining, agriculture, chemistry and food technologies. In these flows, it is very important to know the velocity and flow rate of fluid and the quantity of solid substance. There are many researches on two-phase mixtures of fluid–solid and various mathematical models are developed to examine their steady/unsteady flows.

Badr et al. (2005) have found the amount of solid material in various diameters and various velocity in two-phase liquid–solid flow by building a three-dimensional mathematical model depending on the amount of solid material and liquid velocity in a vertical pipe where there is sudden diameter change. Zhang et al. (2004) have solved the model which they have built for liquid–solid dimensional flow and compared it with experimental results. Chamkha (2000), investigated unsteady laminar flow and heat transfer of a particulate suspension in an electrically conducting liquid through channels and circular pipes in the presence of a uniform transverse magnetic field which is formulated using a two-phase continuum model. The general governing equations of motions are solved in closed form in terms of Fourier cosine and Bessel functions and the energy equations for both phases are solved numerically. Elbashbeshy (2000), studied a viscous incompressible liquid flow along a heat vertical plate, taking into account the variation of the viscosity and thermal diffusivity with temperature in the presence of the magnetic field. The governing equations for laminar free convection of liquid are changed to dimensionless ordinary differential equations by similarity transformation. They are solved by a shooting method numerically. The unsteady laminar boundary layer flow of an electrically conducting liquid past a semi-infinite flat plate with an aligned magnetic field has been studied Takhar et al. (1999). The effect of the induced magnetic field has been included in the analysis. The non-linear

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Nomenclature

$\vec{B}$	induction of magnetic field vector (T)
$\vec{E}$	electrical field intensity vector (V/m)
f_1, f_2	substance quantity of mixture in unit volume ($f_1 + f_2 = 1$)
I_o	modified Bessel function
$\vec{H}$	magnetic filed intensity vector (A/M)
K	interaction coefficient (kgs/m^4)
N	pressure gradient
P	pressure (Pa)
R	radius (m)
r	radial coordinate
s	Laplace operator
t	time (s)
u_1, u_2	axial local velocity component (m/s)
U_1, U_2	Laplace transformation of axial local velocity component (m/s)
$\vec{V}_1, \vec{V}_2$	phase velocity vector (m/s)
z	Axial coordinate
Δ	Laplasian
η	magnetic filed parameter of first phase
μ_1, μ_2	dynamic viscosity (kg/ms)
ρ_1, ρ_2	density (kg/m^3)

partial differential equations have been solved numerically using an implicit finite-difference method. Kuzhir et al. (2005) investigated ferromagnetic suspension flows through a capillary placed between two small strong permanent magnets, the magnetic force acts upon the non-magnetic (silica) particles dispersed in a ferrofluid and they tend to be extruded from the zone of high magnetic field. Particles get concentrated at the entrance section between magnets and form a plug. The increase of hydraulic resistance is due to the relative motion between particulate and ferrofluid phases in the presence of a field. Ishimoto and Kamiyama (1997) studied the effect of nonuniform magnetic field on the linear and nonlinear wave propagation phenomena in two-phase pipe flow of magnetic fluid which is investigated theoretically to realize the effective energy conversion system using boiling two-phase flow of magnetic fluid. The governing equations of two-phase flow are numerically analyzed by using the finite volume method. Sellers and Walker (1999) presented a model for the steady liquid–metal flow through a rectangular duct with electrically insulated walls and with an externally applied, spatially variable, transverse magnetic field.

One of the models examining unstable movements of multi-phase fluids is the model of the movements of the interacting phases. This model is developed by Rahmatulin (1956), improved by Fayzullayev (1966), Latipov (1963) and other scientists. According to this model, each phase of the mixture has local velocity and constant (real or imaginary) physical attributes. The phases interact mutually in a continuous manner. It is assumed that, the phases are homogeneous and evenly distributed per unit volume of the mixture. According to Rahmatulin's model, the following differential equation system explains the unstable movements of the two-phase uncompressible fluids.

$$\rho_1 \frac{d\vec{V}_1}{dt} = -f_1 N + f_1 \mu_1 \Delta \vec{V}_1 + K(\vec{V}_2 - \vec{V}_1); \quad \mathrm{div}\vec{V}_1 = 0 \quad (1)$$

$$\rho_2 \frac{d\vec{V}_2}{dt} = -f_2 N + f_2 \mu_2 \Delta \vec{V}_2 + K(\vec{V}_1 - \vec{V}_2); \quad \mathrm{div}\vec{V}_2 = 0 \quad (2)$$

As seen above, the equation system is obtained by using Navier–Stokes and continuity equations for each phase. Although there are a lot of studies in the literature about the effect of electrical and magnetic fields on single phase conducting flow (Rahmatulin, 1956; Fayzullayev, 1966; Latipov, 1963), this is not the case for the studies about the effect of electrical and magnetic fields on multi-phase flow. It is known that diamagnetic and paramagnetic materials are not affected from magnetic fields. However, ferromagnetic materials are very sensitive to magnetic field. Therefore, to use Rahmatulin model, we need to add the following electromagnetic force expression (Eq. (3)) to Navier–Stokes equation of magnetizable first phase.

$$\eta(\vec{J} \times \vec{H}) \quad \text{or} \quad \vec{J} \times \vec{B} \quad (3)$$

With this additional expression, we will have obtained an equation system to examine the impact of electrical and magnetic fields on non-magnetizable phase via magnetizable phase. As a result, we used the following model in this study:

$$\rho_1 \frac{d\vec{V}_1}{dt} = -f_1 N + f_1 \mu_1 \Delta \vec{V}_1 + K(\vec{V}_2 - \vec{V}_1) + [\vec{J} \times \vec{B}]; \quad \mathrm{div}\vec{V}_1 = 0 \quad (4)$$

$$\rho_2 \frac{d\vec{V}_2}{dt} = -f_2 N + f_2 \mu_2 \Delta \vec{V}_2 + K(\vec{V}_1 - \vec{V}_2); \quad \mathrm{div}\vec{V}_2 = 0 \quad (5)$$

where $\vec{J} = \sigma(\vec{E} + \vec{B} \times \vec{V}_1)$ (6)

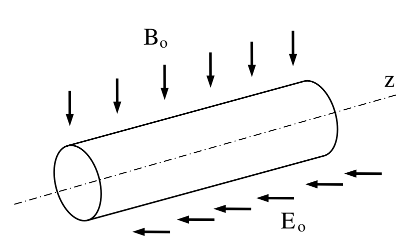

Fig. 1. Schematic diagram of the model.

2. Analytical solution

Consider steady, laminar, fully developed two-phase flow in a horizontal circular pipe due to the action of a constant pressure gradient. Given that, physical parameters, electrical and magnetic field intensity vectors of the phases are constant, i.e. ($\vec{E} = E_0; \vec{B} = B_0$); the vectors $\vec{E}$ and $\vec{B}$ are perpendicular to each other and to the glass pipe (as shown in Fig. 1) for maximum effect, and the flow is one-dimensional, symmetric along the z-axis.

When these values are used at the equations at (4) and (5), the following expressions are obtained:

$$\rho_1 \frac{\partial u_1}{\partial t} = f_1\mu_1 \left[\frac{\partial^2 u_1}{\partial r^2} + \frac{1}{r}\frac{\mathrm{d}u_1}{\mathrm{d}r}\right] + K(u_2 - u_1) + \sigma(E_0 - B_0 u_1)B_0 - f_1 N \tag{7}$$

$$\rho_2 \frac{\partial u_2}{\partial t} = f_2\mu_2 \left[\frac{\partial^2 u_2}{\partial r^2} + \frac{1}{r}\frac{\mathrm{d}u_2}{\mathrm{d}r}\right] + K(u_1 - u_2) - f_2 N \tag{8}$$

where, $u_1 = u_{1z}(r,t) = u_1(r,t)$ and $u_2 = u_{2z}(r,t) = u_2\,(r,t)$

Initial conditions of the problem are selected as follows:

$$u_1(r,0) = 0; \quad u_2(r,0) = 0 \quad \text{at } t = 0 \tag{9}$$

Boundary conditions of the problem are selected as follows:

$$u_1(R,t) = 0; \quad u_1(R,t) = 0 \quad \text{at } r = R \tag{10}$$

The Eqs. (7) and (8) of the model are solved with Laplace method. The following time dependent Laplace transformation equation is used:

$$U(r,s) = U = \int_0^\infty u(r,t)\mathrm{e}^{-st}\mathrm{d}t \tag{11}$$

When this time dependent Laplace transformation equation is applied to Eqs. (7) and (8), the following equations are obtained:

$$\frac{\mathrm{d}^2 U_1}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}U_1}{\mathrm{d}r} - \left(\frac{K+K_1}{f_1\mu_1} + \frac{s}{v_1}\right)U_1 + \frac{K}{f_1\mu_1}U_2 = \frac{NN_1}{s\mu_1} \tag{12}$$

$$\frac{\mathrm{d}^2 U_2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}U_2}{\mathrm{d}r} - \left(\frac{K}{f_2\mu_2} + \frac{s}{v_2}\right)U_2 + \frac{K}{f_2\mu_2}U_1 = \frac{N}{s\mu_2} \tag{13}$$

To simplify the equations above, the following parameters can be used:

$$a = \frac{K+K_1}{f_1\mu_1}; \quad b = \frac{K}{f_1\mu_1}; \quad d = \frac{K}{f_2\mu_2}; \quad m_1 = \frac{N_1}{\mu_1};$$
$$n_1 = \frac{1}{\mu_2}; \quad m_2 = \frac{1}{v_1}; \quad n_2 = \frac{1}{v_2}; \quad N_1 = 1 - \frac{1}{Nf_1}\sigma E_0 B_0;$$
$$K_1 = \sigma B_0^2$$

When these parameters are used in Eqs. (12) and (13), the following equations are obtained.

$$\frac{\mathrm{d}^2 U_1}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}U_1}{\mathrm{d}r} - (a + m_2 s)U_1 + bU_2 = \frac{m_1 N}{s} \tag{14}$$

$$\frac{\mathrm{d}^2 U_2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}U_2}{\mathrm{d}r} - (d + n_2 s)U_2 + \mathrm{d}U_1 = \frac{n_1 N}{s} \tag{15}$$

Then the Laplace transformation for the initial condition values (9) will be as follows:

$$U_1(r,0) = 0; \quad U_2(r,0) = 0 \tag{16}$$

The Laplace transformation for the boundary condition values (10) will be as follows:

$$U_1(R,s) = 0; \quad U_2(R,s) = 0 \tag{17}$$

We chose D'Alambert method for the solution of simplified Equations at (14) and (15). To solve the equation, by multiplying both sides of the Eq. (14) with A and adding it to Eq. (15) side by side. If we choose A as in the following expression,

$$A = \frac{(a + m_2 s)A - d}{d + n_2 s - bA} \tag{18}$$

The Eq. (18) is solved as follows:

$$A_{1,2} = \frac{-[a - d + s(m_2 - n_2)] \pm \sqrt{[a - d + s(m_2 - n_2)]^2 + 4db}}{2b} \tag{19}$$

Then the following equation is obtained:

$$\frac{\partial^2}{\partial r^2}(AU_1 + U_2) + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(AU_1 + U_2) - (d + n_2 s - bA)[AU_1 + U_2] = \frac{N}{s}(Am_1 + n_1) \tag{20}$$

The general solution of this equation is:

$$AU_1 + U_2 = C_1 I_0(\sqrt{d + n_2 s - bA})r + C_2 K_0 \times (\sqrt{d + n_2 s - bA})r - \frac{\frac{N}{s}(Am_1 + n_1)}{d + n_2 s - bA} \tag{21}$$

According to boundary conditions Eq. (17), $C_2 = 0$ and therefore C_1 will be as follows:

$$C_1 = -\frac{\frac{N}{s}(Am_1 + n_1)}{d + n_2 s - bA} \tag{22}$$

Then the general solution of the Eq. (21) will be as follows:

$$AU_1 + U_2 = \frac{N(Am_1 + n_1)}{s(d + n_2 s - bA)}\left[\frac{I_0(\sqrt{d + n_2 s - bA})r}{I_0(\sqrt{d + n_2 s - bA})R} - 1\right] \tag{23}$$

When we replace the roots of A_1 and A_2 with A in Eq. (23), the following is obtained:

$$A_1 U_1 + U_2 = \frac{N(A_1 m_1 + n_1)}{s(d + n_2 s - bA_1)}\left[\frac{I_0(\sqrt{d + n_2 s - bA_1})r}{I_0(\sqrt{d + n_2 s - bA_1})R} - 1\right] \tag{24}$$

$$A_2 U_1 + U_2 = \frac{N(A_2 m_1 + n_1)}{s(d + n_2 s - bA_2)}\left[\frac{I_0(\sqrt{d + n_2 s - bA_2})r}{I_0(\sqrt{d + n_2 s - bA_2})R} - 1\right] \tag{25}$$

Then by using Eqs. (24) and (25), the Laplace transformations of the flow rates of the phases are found as follows:

$$U_1 = \frac{N}{s(A_1 - A_2)}\left[\frac{A_1 m_1 + n_1}{D_1^2}\left(\frac{I_0(D_1 r)}{I_0(D_1 R)} - 1\right) - \frac{A_2 m_1 + n_1}{D_2^2}\left(\frac{I_0(D_2 r)}{I_0(D_2 R)} - 1\right)\right] \quad (26)$$

$$U_2 = \frac{N}{s(A_2 - A_1)}\left[A_2\frac{A_1 m_1 + n_1}{D_1^2}\left(\frac{I_0(D_1 r)}{I_0(D_1 R)} - 1\right) - A_1\frac{A_2 m_1 + n_1}{D_2^2}\left(\frac{I_0(D_2 r)}{I_0(D_2 R)} - 1\right)\right] \quad (27)$$

where, $D_1 = \sqrt{d + n_2 s - bA_1}$ and $D_2 = \sqrt{d + n_2 s - bA_2}$

By using Reverse Laplace transformation approach, the coordinate based expressions for the unsteady state of phases is found. If we want to use these equations for the steady state flow, local velocity expression for each phase will be as follows:

$$u_1(r) = \frac{N}{A_{1k} - A_{2k}}\left[\frac{A_{1k} m_1 + n_1}{(M_{1k})^2}\left(\frac{I_0(M_{1k})r}{I_0(M_{1k})R} - 1\right) - \frac{A_{2k} m_1 + n_1}{(M_{2k})^2}\left(\frac{I_0(M_{2k})r}{I_0(M_{2k})R} - 1\right)\right] \quad (28)$$

$$u_2(r) = \frac{N}{A_{2k} - A_{1k}}\left[A_{2k}\frac{A_{1k} m_1 + n_1}{(M_{1k})^2}\left(\frac{I_0(M_{1k})r}{I_0(M_{1k})R} - 1\right) - A_{1k}\frac{A_{2k} m_1 + n_1}{(M_{2k})^2}\left(\frac{I_0(M_{2k})r}{I_0(M_{2k})R} - 1\right)\right] \quad (29)$$

To simplify the equations, following terms are used.

$$A_{1k,2k} = \frac{-[a - d] \pm \sqrt{[a - d]^2 + 4db}}{2b}$$

$$M_{1k} = \sqrt{d - bA_{1k}} = \sqrt{0.5\left(a + d - \sqrt{(a - d)^2 + 4bd}\right)} \quad \text{and}$$

$$M_{2k} = \sqrt{d - bA_{2k}} = \sqrt{0.5\left(a + d + \sqrt{(a - d)^2 + 4bd}\right)}$$

The velocity profiles of two-phases according to the calculations based on Eqs. (28) and (29) are shown in Figs. 2–5.

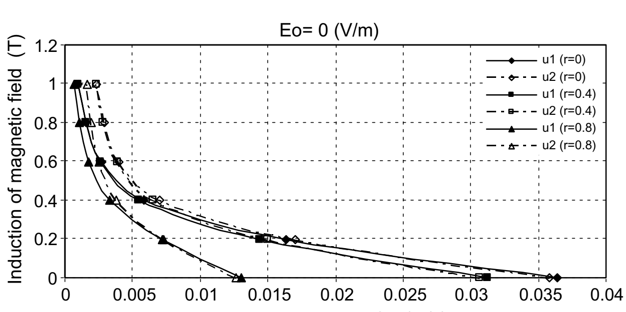

Fig. 2. The effect of the magnetic field on the local velocity of the phases along the pipe radius at selected points.

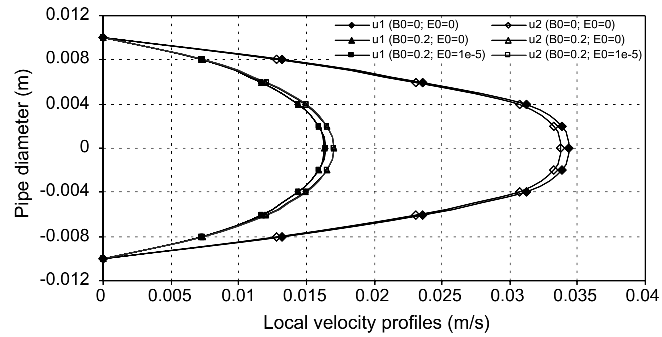

Fig. 3. The effect of the electrical and magnetic fields on the flow along the pipe diameter at different B_0 and E_0.

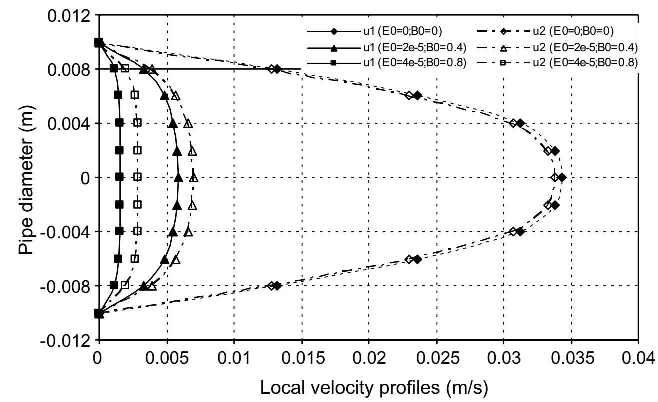

Fig. 4. The local velocity profiles of the phases along the pipe diameter at different B_0 and E_0.

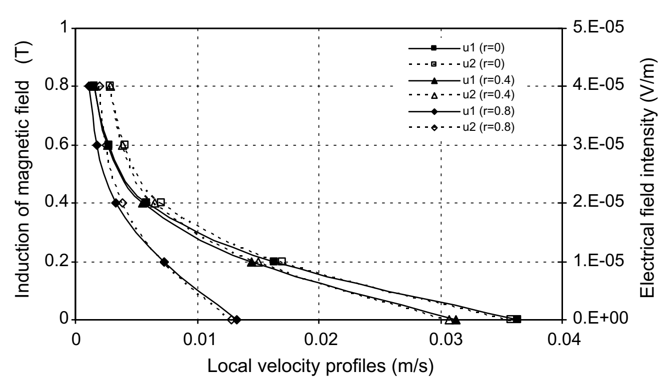

Fig. 5. The effect of the electrical and magnetic field on the flow along the pipe radius at selected points.

The specifications of the materials used in calculations are as follows: The fluid with subscript 1 is iron powder ($f_1 = 0.3$, $\mu_1 = 1.10^{-5}$ kg/ms) and the fluid with subscript 2 is pure water ($f_2 = 0.7$, $\mu_2 = 10^{-3}$ kg/ms). Also, the following values are used for calculations: Interaction coefficient, $K = 520\ \mathrm{kgs/m^4}$, pressure gradient, $N = \frac{\partial P}{\partial z} = -1$ Pa/m, pipe radius, $R = 0.01$ m, electrical conductivity of iron, $\sigma = 10^3 \frac{1}{\mathrm{Ohm\ m}}$, magnetization coefficient, $\eta = 1000$, induction of magnetic field, $B_0 = 0$–1 T, electrical field intensity, $E_0 = 0 - 4 \times 10^{-5}$ V/m.

3. Result and discussion

In this study, the magnetic and electrical field effects of the two-phase flow steady flows in the circular glass pipe are investigated theoretically. The Navier–Stokes equations of two-phase flow is analytically solved by using the methods of Laplace transformation and D'Alambert. By using the analytical solution, local velocity profile expressions for each phase are obtained in terms of electrical field intensity, magnetic field induction, time, coordinates, physical parameters of the fluids like density, dynamic viscosity and electrical conductivity. Moreover, expressions for the local velocity profiles of both phases are derived. Then the effect of the electrical and magnetic field on the two-phase steady state flow is examined in detail. The local velocity profile of the phases are found with the following values: The physical attributes of the fluids, radius of the pipe, pressure gradient are constant; electrical field intensity E_0 takes values between 0 and 4×10^{-5} V/m with 1×10^{-5} V/m step size; magnetic field induction B_0 takes values between 0 and 1 T with 0.2 T step size. The results of the analytical solution are shown in Figs. 2–5.

As it is also conformed by Eq. (1), electrical field itself cannot have any effect on the momentum of the flow since it cannot form a force by itself. Therefore, the effect of different intensity levels of the magnetic field on the fluid flow when electrical field intensity is zero is shown in Fig. 2. As seen from the figure, along the pipe radius ($r = 0$, $r = 0.004$ and $r = 0.008$ m) increasing magnetic field intensity causes the local velocity of the flow to decrease. While there is more decrease in the velocity at the first values of the applied magnetic field intensity, increasing magnetic field intensity causes less decrease in the velocity. The amount of decrease when $B_0 = 1$ T at $r = 0$ point is found as 97% for the first phase, 93% for the second phase; at $r = 0.004$ m point it is found as 96% for the first phase, 92% for the second phase and at $r = 0.008$ m point it is found as 94% for the first phase, 87% for the second phase. As it is also seen from these values, the decrease of the local velocity of the phases is higher at the center of the pipe and it gets smaller along the pipe radius towards the wall of the pipe. The cause of the decrease in the local velocity of the first phase is the magnetic force against the flow which is caused by the magnetic field perpendicular to the flow. The cause of the decrease in the local velocity of the second phase is the interaction with the first phase. In other words, magnetic field will affect the flow of second phase which has low conductivity and is not magnetizable, via the first phase which has high conductivity and is magnetizable.

The Figs. 3–5 show how the electrical field and magnetic field which are perpendicular to each other and to the flow, affect the flow. Fig. 3 shows the local velocity profiles of the phases where B_0 is constant and E_0 changes. Fig. 4 shows the local velocity profiles of the phases where both B_0 and E_0 changes. Fig. 5 shows the local velocity profiles of the phases at some selected points where both B_0 and E_0 changes.

The figures clearly show the decrease in the local velocity values of the flow of the phases due to the simultaneous application of the electrical and magnetic fields. When calculations are done according to the curves of Fig. 3 to find the change from $B_0 = 0$ T and $E_0 = 0$ V/m to $B_0 = 0.2$ T and $E_0 = 1 \times 10^{-5}$ V/m, it is found that the decrease of the local velocity for the first phase is 52.4% and for the second phase it is 49.5%. The change from $B_0 = 0.2$ T, $E_0 = 1 \times 10^{-5}$ V/m to $B_0 = 0.2$ T, $E_0 = 0$ V/m causes a difference of 0.18% at the local velocity profiles of the phases. Therefore, the curves of these states overlap at the figure. When B_0 and E_0 are doubled from the values of Fig. 3, which was $B_0 = 0.2$ T, $E_0 = 1 \times 10^{-5}$ V/m, the local velocity of the first phase decreases by an additional 12.37% and the local velocity of the second phase decreases by an additional 12.23% (approximately 35%) as seen at Fig. 4.

To be able to see the effect of both magnetic and electrical field together on the flow, the variation of the local velocity profiles of the phases in relation to pipe radius (at the points $r = 0$, $r = 0.004$ and $r = 0.008$ m), magnetic field induction and electrical field intensity are shown at Fig. 5. As seen from the figure, there is decrease in the local velocity profile of the flow of the phases even when it is exposed to both electrical and magnetic fields simultaneously. When $B_0 = 0.8$ T and $E_0 = 4.10^{-5}$ V/m, at the $r = 0$ point the decrease is 95.6% for the first phase, 91.83% for the second phase; at the $r = 0.004$ m point the decrease is 95% for the first phase, 90.75% for the second phase and at the $r = 0.008$ m point the decrease is 91.65% for the first phase, 84.76% for the second phase. As mentioned above, the decrease of the local velocity of the phases is higher at the center of the pipe and it gets smaller along the pipe radius towards the wall of the pipe.

4. Conclusion

In this paper, the effect of the electrical and magnetic field which is applied perpendicular to the flow and each other on the two-phase steady flow in the circular glass pipe has been investigated. The governing equations for this investigation was derived and solved analytically. For the two-phase flow, where first one has high electrical conductivity and it is magnetizable (first phase) and second one has low electrical conductivity and it is not magnetizable (second phase); the following conclusion is obtained:

(i) When only the magnetic field is applied perpendicular to the flow of the mixture, local flow velocity of the first phase is decreased due to the direct effect of the magnetic field; and local flow velocity of the second phase is decreased due to the indirect effect of the magnetic field which is caused by the interaction of the phases.

(ii) When the magnetic and electrical fields perpendicular to each other and to the flow are applied, the local flow velocity of the first phase is decreased due to the direct effect and the local flow velocity of the second phase is decreased due to the indirect effect. This decreases in the local velocity of the phases are dependent on electrical field direction.

(iii) When only electrical field is applied, none of the phases is affected.

As a result, it is seen that the electromagnetic force is effecting the nonconductor phase of the mixture through the conductor phase which it can directly affect.

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